Statistics between mainshocks and foreshocks in Italy and Southern California

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[1] The most used and accepted models for daily forecasts are based on short-term space and time earthquake clustering for occurrence rates and on the Gutenberg-Richter law for the frequency-magnitude. These models have been demonstrated to produce reliable prospective space-time-magnitude forecasts during an aftershock sequence, but their skill in forecasting mainshocks is still under discussion. This paper studies the foreshock statistics of the Italian and Californian seismicity in two ways: i) we compare the foreshock activity observed in real seismic catalogs and in synthetic catalogs derived from a pure Epidemic-Type Aftershock Sequence (ETAS) model; ii) we analyze the triggering capability of earthquakes using different ETAS parameterizations, in order to check whether large events are triggered in the same way as regular earthquakes. The results indicate that the foreshock activity observed in the real catalogs is compatible with what is expected by the ETAS model. Moreover, we find that the empirical foreshock rates have an intrinsic variability due to limited sampling that may explain most of the differences found so far in different seismic catalogs.


1. Introduction

[2] One of the auspicious future developments of operational earthquake forecasting is the use of short-term forecasting models to help society to brace for a potentially destructive earthquake (T. H. Jordan et al., Operational earthquake forecasting: State of knowledge and guidelines for implementation, submitted to Annales Geophysicae, 2011). At present, the most used and accepted models for daily forecasts are based on short-term space and time earthquake clustering and on the Gutenberg-Richter frequency-magnitude law [e.g., Kagan and Knopoff, 1987; Ogata, 1988; Reasenberg and Jones, 1989; Zhuang et al., 2002; Gerstenberger et al., 2005]. These models have been demonstrated to produce reliable prospective space-time-magnitude forecasts during an aftershock sequence [Marzocchi and Lombardi, 2009] and also retrospective comparative tests have shown that these models can be used as a reasonable null hypothesis to test future forecasting and/or predictions models [Woessner et al., 2011].

[3] Nonetheless, it has been often argued whether these models are able to provide reliable forecasts for mainshocks and/or large events; in other words, whether the seismicity observed before a large shock is compatible to what is expected by an Epidemic-type Aftershock Sequence (ETAS) model [Ogata, 1988], and therefore similar to what happen before any earthquake regardless its magnitude. Some researchers [e.g., Helmstetter and Sornette, 2003; Felzer et al., 2004; Zhuang et al., 2008; Christophersen and Smith, 2008] have already given a partial answer to this question showing that some aspects of foreshocks occurrences (according to different plausible definitions) are not different from what are expected by ETAS model. In particular, using different approaches they show that the space-time clustering before an earthquake appears independent from its magnitude.

[4] On the other hand, many other researchers think that a large earthquake may have a different preparatory phase characterized by some peculiar precursory seismicity [e.g., Mignan et al., 2006; Wu et al., 2008]. This general thinking stands behind all short-term prediction models, including Agnew and Jones’s [1991, hereinafter AJ91] model that is currently used by the California Earthquake Prediction Evaluation Committee (CEPEC) for short-term operational earthquake forecasting in California [Michael, 2011]. In particular, AJ91’s model identifies empirically foreshocks in a specified space ($S = 10$ km), time ($T = 3$ days), and magnitude ($M = 3$ or less degrees from the main event) window. The premise is that some earthquakes are effectively foreshocks - i.e., distinguishable in a prospective operational point of view from other earthquakes - and therefore potentially very helpful to improve our operational forecasting of large events. We argue that AJ91’s model is expected to be more reliable and skilled than ETAS models in the cases where there really exists a peculiar seismic precursory phase before large shocks in this small T-S window. Otherwise, the results of AJ91’s model should converge to the ETAS forecasts, being the empirical foreshock statistics equal to what expected by the ETAS model. Other differences of AJ91’s and ETAS forecasts may arise from the use of a different frequency-magnitude relationship. This issue has been reported by Michael and Jones [1998], and Michael [2011] and we do not investigate it in this study.

[5] Here, we take a fresh look at this problem by applying two different types of analysis to the Italian and the Southern Californian seismic catalogs. In the first analysis, we compare the foreshock rates observed in real and ETAS-simulated catalogs, keeping the foreshock definition and space-time window similar to AJ91. The use of such a space-time-magnitude window with sharp boundaries deserves a justification. The primary rationale behind this choice is the clear operational usefulness [e.g., Agnew and Jones, 1991; van Stiphout et al., 2010]. From a scientific point of view, it is obvious that the use of sharp boundaries
may lead to bias in describing a process that does not have such natural sharp cutoff; anyway, we argue that this is not a problem in comparing earthquake statistics, since sampling always implies the definition of windows with sharp boundaries. Noteworthy, this space-time window before large earthquakes has never been specifically considered in previous analysis [cf. Helmstetter and Sornette, 2003; Felzer et al., 2004; Zhuang et al., 2008; Christophersen and Smith, 2008]. In fact, previous works are largely based on the foreshock activity of small-to-medium magnitude earthquakes, and look at different space-time-magnitude windows compared to the one considered here. In this respect, our analysis is meant to provide a complementary contribution, more focused on an operational use, to this important argument. In the second analysis, we apply different parameterizations of ETAS model to the real seismicity in order to check if the triggering contribution for large and small earthquakes is different. Note that in this case, it is not necessary to define what a foreshock is.

2. The Seismic Catalogs

[6] As regards Italy, we have used the CPTI08 catalog. This catalog is a pre-release version of the CPTI catalog family [see Gruppo di Lavoro CPTI, 2004], which was prepared for the Collaboratory for the Studies of Earthquake Predictability (CSEP) project [Marzocchi et al., 2010] and covers the period 1901–2006. The catalog is available in its original form together with a description at http://www.data.scec.org/) catalog from 1932-01-01 to 2007-01-31, as described by Hutton et al. [2010]. For fitting the ETAS model, we choose a magnitude threshold of $M_w$ = 4.5. Such a threshold guarantees to have a more or less constant seismic rate since 1950 and a linear Gutenberg–Richter law (see Figure 1).

[7] The second dataset is the SCEDC (Southern California Earthquake Data Center, http://www.data.scec.org/) catalog for fitting the ETAS model, we choose a magnitude threshold of $M_w$ = 4.5. Above which the catalog is believed complete except 1 or 2 huge aftershock sequences. The area considered is the same as that of Zhuang et al. [2008].

3. Empirical Analysis: The Foreshock Rates in Italy and Southern California

[8] The definition of “foreshock” is necessarily arbitrary until a causative process linking these specific earthquakes with a large event is found. Here, we define foreshock and mainshock adopting an operational point of view similar to AJ91. Mainshocks ($M$) are all events with magnitude in the range $M \geq M^\alpha$. Each earthquake with magnitude in the range $[M_{min}, M^\alpha]$ is a potential foreshock ($F$). Retrospectively, we identify a foreshock $F$ as a $F$ with a distance $\Delta$ from the future mainshock less than 10 km and a time to the mainshock $\tau$ less than 3 days. We set $M_{min} = 3.95$. $M_{min}$ (the minimum value for $F$) has been chosen for two reasons: i) it guarantees to have a sufficient completeness of the seismic catalogs back in time to calculate foreshock rates; ii) such
felt earthquakes raise a natural concern among the people, and therefore they have a particular importance from an operational point of view. \( M^* \) is set for both catalogs to 5.45 because it is a magnitude that may cause significant damages in Italy. The largest number of earthquakes in California allows us to consider also the case of \( M^* = 5.95 \) with the purpose to check the consistency of the results for a different definition of target event. From this counting we can calculate two different foreshock rates. First, we calculate the rate between \( F(M) \) and the total number of \( M \). Reasenberg [1999] called this “foreshock rate”, but we find this definition rather ambiguous and we prefer to name it as foreshock-mainshock (\( \Pi_1 \)) rate. Second, we calculate the rate between \( F \) and \( \hat{F} \) (\( \Pi_2 \)) rate. This number gives the probability that an observed \( \hat{F} \) would turn out to be a foreshock, i.e., \( \Pi_2 = P(\hat{F}|\hat{F}) \).

The empirical counting of \( M \), \( F(M) \), and \( \hat{F} \) is reported in Table 1. In Italy, we count 6 \( F(M) \), 26 \( M \), and 742 \( \hat{F} \). \( \Pi_1 \) rate is 6/26 = 0.23. In order to compare this rate with similar analysis, Reasenberg [1999] proposed to normalize this ratio per the magnitude aperture, i.e., the amplitude of the foreshock magnitude (here, 1.5). Therefore the \( \Pi_1 \) rate per unit magnitude is 0.15. This ratio is comparable to foreshock rates in other areas of the world [e.g., Reasenberg, 1999]. Console et al. [1999] found smaller values for \( \Pi_1 \) in Italy, but their estimations are based on a declustered catalog, and therefore the results cannot be easily compared. The \( \Pi_2 \) rate is about 0.008. In Southern California, we count 28 \( F(M) \), 89 \( M \), and 2163 \( \hat{F} \). The \( \Pi_1 \) rate is 28/89 = 0.31 and the \( \Pi_1 \) rate per unit magnitude is 0.21; this figure seems to be significantly larger than what reported by Reasenberg [1999]. The \( \Pi_2 \) rate is about 0.013.

[10] In order to check if the observed number of \( M \), \( F(M) \), and \( \hat{F} \) are compatible with the ETAS model, we run the same counting on ETAS synthetic catalogs. Specifically, we produce 1000 synthetic catalogs with the same length as the Italian and Southern Californian seismic catalogs. The simulation algorithm is based on that of Zhuang et al. [2004]: (1) the background events are generated as a non-homogeneous Poisson process with rate \( \mu(x, y) \), a function of spatial locations \((x, y)\) but constant in time, which is obtained by smoothing the outputs of applying stochastic declustering method [Zhuang et al., 2002] to the original catalogs; (2) each event, say, \((t_i, x_i, y_i, m_i)\), once being generated, triggers its own \( N_i \) children, where \( N_i \) is a Poisson random variable with a mean of \( k(m_i) = A \exp(\alpha(m_i - m_0)) \); (3) the occurrence times and locations of the children from event \((t_i, x_i, y_i, m_i)\) have, respectively, a probability density function of \( g(t|t_i) = (p-1)\exp[\alpha(m_i - m_0)] / c, t \geq t_i \), and \( f(x,y|x_i,y_i,m_i) = \frac{q-1}{\pi D^2 c^q(m_i - m_0)} \left[ 1 + \frac{(x-x_i)^2 + (y-y_i)^2}{D^2 c^q(m_i - m_0)} \right]^{-(q-1)}, \)

\( m_0 \) being the magnitude threshold; and, (4) the magnitudes for all the events are generated by the Gutenberg–Richter law. The parameters of the ETAS model are obtained through the maximum likelihood estimation of the seismic catalogs described above. The parameters are \( A = 0.121, c = 0.0625 \) day, \( \alpha = 2.167, p = 1.264, D^2 = 7.78 \times 10^{-4} \) deg², \( q = 1.600, \)

<table>
<thead>
<tr>
<th>Dataset</th>
<th>( M^* )</th>
<th>#( F(M) )</th>
<th>#( M )</th>
<th>#( \hat{F} )</th>
<th>( \Pi_1 )</th>
<th>( \Pi_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Italian catalog</td>
<td>5.45</td>
<td>6</td>
<td>26</td>
<td>742</td>
<td>0.23</td>
<td>0.008</td>
</tr>
<tr>
<td>Southern California catalog</td>
<td>5.45</td>
<td>28</td>
<td>89</td>
<td>2163</td>
<td>0.31</td>
<td>0.013</td>
</tr>
<tr>
<td>Southern California catalog</td>
<td>5.95</td>
<td>5</td>
<td>28</td>
<td>2212</td>
<td>0.18</td>
<td>0.0023</td>
</tr>
</tbody>
</table>

**Figure 2.** Cumulative curves of \( M \), \( F(M) \), and \( \hat{F} \) for the Italian and Californian seismic catalogs. The vertical red dashed lines represent the observed values.
4. ETAS Modeling: Triggering of Small and Large Earthquakes in Italy and Southern California

The ETAS model [see, e.g., Ogata, 1988; Zhuang et al., 2002] forecast the future seismicity estimating the conditional intensity

$$\lambda(t, x, y) = \mu(x, y) + \sum_{i \leq t} k(m_i)g(t|t_i)f(x, y|x_i, y_i, m_i)$$  \hspace{1cm} (2)

where $t, x,$ and $y$ are the time and spatial coordinates, and the functions $k()$, $g()$ and $f()$ describe how past earthquakes modify the intensity as a function of their magnitude, elapsed time and spatial coordinates, respectively, all taking the same forms as in the simulation algorithm in Section 3. We use the ETAS model in two different ways. First, we fit regular ETAS model, namely, model 1, to each earthquakes above the completeness magnitude. Then, we fit the adjusted model [see Zhuang et al., 2008], namely, model 2, to earthquakes in $M$. That is, the conditional intensity is calculated for each earthquake with magnitude equal to or larger than $M^*$, but allowing triggering effect as from events between $M^*$ and the completeness threshold. After fitting the two models to the two real seismic catalogs, we use the stochastic declustering method to obtain the probability that each $M$ is triggered by previous potential foreshock events, i.e., for an $M$, namely, the $n$th event in the catalog,

$$\rho_j^{(n)}(m) = \frac{\sum_{i \leq t \leq n} k^{(j)}(m_i)g^{(j)}(t_{i|t_i})f^{(j)}(x_{i|y_i}|x_i, y_i, m_i)1(m_i < M^*)}{\lambda^{(j)}(t_{i|t_i}, x_i, y_i)}$$  \hspace{1cm} (3)

Figure 3. Distribution of the differences $\rho_j^{(2)} - \rho_j^{(1)}$ (model 2 - model 1; see equation (5) in the text) of the mainshocks triggering probabilities for the Italian and Californian catalogs. (top) The cumulative distributions for the real (solid blue line) and one synthetic (solid green line) catalogs. (bottom) The histograms for the real seismic catalogs.
where \( j \) is 1 (model 1) or 2 (model 2). If \( \rho_f^{(1)} \sim \rho_f^{(2)} \) for each large event, this means that the earthquakes with magnitude above \( M^* \) are a random sample of all earthquakes, as implicitly assumed by the ETAS model. If larger earthquakes are triggered differently from smaller events, we expect that \( \rho_f^{(2)} \neq \rho_f^{(1)} \). Of particular interest is the case \( \rho_f^{(2)} > \rho_f^{(1)} \) indicating that large earthquakes are more triggered by small earthquakes compared to earthquakes of smaller magnitudes.

[12] In Figure 3, we report the results for the real Italian and Southern Californian catalogs. For the sake of comparison, we add the cumulative distribution of one synthetic catalog. The lack of a theoretical distribution for \( \rho_f^{(2)} \) and the prohibitive computation time to get the empirical distribution from synthetic catalogs prevent us to formally test the statistical significance of the differences found for the real catalogs. Anyway, it is worth noting that about 85% of the differences are < 0.1. This stands for a negligible difference for most of the mainshocks. We cannot exclude that some mainshocks have a peculiar triggering. For example, we note that the largest difference is relative to the Colfiorito earthquake, September 1997; previous papers have already found that the occurrence of this sequence cannot be explained by an ETAS model [Lombardi et al., 2010] since it has been significantly affected by fluids intrusion [e.g., Miller et al., 2004]. Our results indicate that this kind of events seem to be rare, and in most of the cases large events are triggered as smaller events.

5. Final Remarks

[13] In this paper we have analyzed the foreshock activity in the Italian and Southern Californian seismic catalogs by comparing real observations with ETAS-simulated catalogs and by using two different kinds of ETAS parameterizations. The goal is to look for potential discrepancies between the foreshock activity of the real catalogs and of the ETAS model. The result obtained indicate that, acknowledging the limited extension in time of the catalogs, ETAS model seems to describe quite well the observed foreshock activity. In particular, i) large earthquakes appear to be triggered as any other earthquakes, and ii) according to a specific operational definition of foreshock, ETAS synthetic catalogs provide a number of potential foreshocks, mainshocks, and mainshocks anticipated by foreshocks that are compatible with the observations. The ETAS synthetic catalogs show a large variability of the foreshock rates \( \Pi_1 \) and \( \Pi_2 \). This poses important constraints in evaluating the differences among distinct tectonic areas, and in using the empirical rates for large earthquake forecasting.

[14] The consistency between model and observations has important practical consequences. In fact, ETAS model has been proposed to forecast aftershocks notwithstanding, our results extend the possible use of the ETAS model to track the probabilistic evolution of seismic sequences that might anticipate large events. During a seismic sequence, ETAS model may provide high probability gain compared to the background activity [e.g., Reasenberg, 1999; Jordan et al., submitted manuscript, 2011], but its implicit features (i.e., the use of the Gutenberg–Richter law) prevent getting probabilities larger than 1% most of the times. This property poses constraints on its potential societal utility even though some seismic risk mitigation actions might be planned after all [van Stiphout et al., 2010; Jordan and Jones, 2010; Woo, 2010].

[15] Please note that, our results are not meant to prove that we live in an “ETAS world”, but only that this model represents a reasonable and reliable picture of the present state of knowledge about the short-term forecasting of large earthquakes.

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References


Conole, R., F. Di Luccio, M. Murr, M. Imoto, and G. Stavarakakis (1999), Short term and short range patterns in different seismic areas of the world, Nat. Hazards, 19, 107–121.


