Spatiotemporal correlations of aftershock sequences

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[1] Aftershock sequences are of particular interest in seismic research because they may condition seismic activity in a given region over long time spans. Although they are typically identified with periods of enhanced seismic activity after a large earthquake as characterized by the Omori law, our knowledge of the spatiotemporal correlations between events in an aftershock sequence is limited. Here, we study the spatiotemporal correlations of two aftershock sequences form California (Parkfield and Hector Mine) using the recently introduced concept of “recurrent” events. We find that both sequences have very similar properties and that most of them are captured by the space-time epidemic-type aftershock sequence (ETAS) model if one takes into account catalog incompleteness. However, the stochastic ETAS model does not capture those spatiotemporal correlations that give rise to the observed distribution of recurrent events on small spatial scales. We also find that there is no clear evidence for stress shadows that have been observed for smaller earthquakes.


1. Introduction

[2] One of the grand challenges for seismology is to establish the relationship between stress and strain in the lithosphere [Forsyth et al., 2009]. While motions of tectonic plates and surface deformations can be measured precisely with satellite imaging and networks of Global Positioning System receivers, strainmeters, seismometers and tiltmeters, the causative stress can only be inferred. However, the temporally and spatially dependent rheology which describes the linkage between the forces (stresses) and the resulting deformations (strains) is generally not well understood [Kanamori and Brodsky, 2004]. This makes it currently impossible to conclusively answer how some earthquakes trigger other earthquakes thousands of kilometers away as well as to predict earthquakes reliably, to give a few examples.

[3] An alternative approach to gain insight into the underlying dynamics of earthquakes is to study the spatiotemporal patterns of seismicity where each earthquake is treated as a point event in space and time with a given magnitude. Such an approach may shed light on the fundamental physics since these patterns are emergent processes of the underlying many body nonlinear system. Indeed, it has been proved successful in many cases and led to the discovery of new key features of seismicity [Rundle et al., 2003; Corral, 2004; Davidsen and Goltz, 2004; Shcherbakov et al., 2004; Davidsen and Paczuski, 2005; Batesi and Paczuski, 2005; Felzer and Brodsky, 2006; Hainzl et al., 2006; Corral, 2006; Marsan and Lengliné, 2008; Zaliapin et al., 2008]. For example, Davidsen et al. [2006, 2008] found that the spatiotemporal clustering of earthquakes in southern California shows nontrivial features in the sense that these features are not a consequence of independent spatial and temporal properties of seismicity. This has led to a new and independent estimate of the rupture length and its scaling with magnitude. In particular, the results provided further evidence for a shadowing effect associated with smaller earthquakes [Rubin, 2002; Fischer and Horálek, 2005; Hainzl and Marsan, 2008]. The key to these findings was a unique approach to quantify nontrivial spatiotemporal clustering based on the view that any suitable definition of clustering should be purely contextual and depend only on the actual history of events without any further assumptions [Davidsen et al., 2008]. This approach utilizes the notion of space-time records to define “recurrences” and maps seismicity onto a graph or network, thus, allowing the characterization of spatiotemporal clustering by means of tools from complex network theory [Albert and Barabasi, 2002; Newman, 2003].

[4] To elucidate the origin of the observed nontrivial clustering further, we study here the spatiotemporal correlations of aftershock sequences which follow large shallow earthquakes. Aftershocks are the most obvious example of earthquakes that are triggered in part by preceding events as follows from the observed increased seismic activity captured by the Omori law [Utsu et al., 1995]. Thus, their specific clustering in space and time should provide information on the underlying dynamics and triggering mechanisms [see, e.g., Felzer and Brodsky, 2006; Gomberg and Felzer, 2008]. Moreover, aftershocks are important from a conceptual point of view since the current main paradigm in statistical seismology classifies earthquakes as triggered events like aftershocks and “background” events which are hypothesized to...
be induced by other means. It is important to realize, however, that the notion of background events, and aftershocks, for that matter, is not well defined as no clear physical difference between such events has been established. In particular, background events could be artifacts to a large extent if small earthquakes can trigger larger events [Helmstetter et al., 2006; Marsan and Lengliné, 2008] since many small earthquakes are typically not detected [Sornette and Werner, 2005a, 2005b].

For the aftershock sequences associated with the Parkfield earthquake (28 September 2004) and the Hector Mine earthquake (16 October 1999), we find that both sequences show very similar spatiotemporal correlations as quantified by the method of Davidsen et al. [2006, 2008]. While most features are also similar to what has been observed previously for a 20 year catalog from southern California [Davidsen et al., 2008], there is no indication of a shadowing effect. Moreover, we find that most of the observed features can be captured by a space-time version of the epidemic-type aftershock sequence (ETAS) model [Helmstetter and Sornette, 2002a; Ogata and Zhuang, 2006] if catalog incompletenesses [Lennartz et al., 2008] is taken into account. This suggests that the spatiotemporal correlations are to a large extent a consequence of a few established laws of seismicity and missing data. This was further confirmed by a comparison with a simple nonhomogeneous Poisson (NHP) model following the Omori law, but without any spatiotemporal correlations between events. Yet, we find that the considered ETAS model has several shortcomings with respect to the spatial distribution of seismicity.

2. Aftershock Sequences and the ETAS Model

Although there is considerable statistical variability associated with aftershocks, their behavior appears to satisfy several scaling laws to a reasonably good approximation. Among them are the Gutenberg-Richter scaling relation for the frequency-magnitude distribution [Gutenberg and Richter, 1949] which is certainly satisfied on long timescales [Shcherbakov et al., 2006]. Båth's law for the difference between the magnitudes of a main shock and its largest aftershock [Båth, 1965] as well as the modified Omori law for the temporal decay of aftershock rates [Utsu et al., 1995; Shcherbakov et al., 2004; Hainzl and Marsan, 2008; Narteau et al., 2009]. More recently, another scaling law characterizing the epicenter distribution of aftershocks has been found [Felzer and Brodsky, 2006].

In an attempt to establish a statistical null model of aftershocks which would incorporate some of these scaling laws, the epidemic-type aftershock sequence (ETAS) model was introduced [Kagan and Knopoff, 1987; Ogata, 1988; Helmstetter and Sornette, 2002b]. This model describes a stochastic branching process in which any earthquake may trigger other earthquakes, which in turn may trigger more, and so on. In particular, the location and the time of occurrence of each “daughter” event is strongly correlated to its “mother” event. The occurrence rate of daughter events at time $t$ and position $\vec{r}$ triggered by a mother event of magnitude $m_i$ at time $t_i$ and position $\vec{r}_i$, is defined as

$$\phi_m(t - t_i, \vec{r} - \vec{r}_i) = \rho(m_i) \Psi(t - t_i) \Phi_m(\vec{r} - \vec{r}_i),$$

where $\rho(m_i)$ is the average number of aftershocks directly triggered by an event of magnitude $m_i$, $\Psi(t - t_i)$ is the normalized temporal distribution of directly triggered events and $\Phi_m(\vec{r} - \vec{r}_i)$ is the normalized spatial distribution of directly triggered events in two dimensions. $\rho(m_i)$ is assumed to follow the productivity law

$$\rho(m_i) = K \Omega^{\alpha(m_0 - m_i)},$$

where $m_i > m_0$ and $K$, $\alpha$ are constants. Note that the productivity law is zero below $m_0$ implying that events smaller than $m_0$ do not trigger other earthquakes. This condition is necessary to ensure a finite total occurrence rate (see Sornette and Werner [2005a] for a discussion of its physical relevance). $\Psi(t)$ is assumed to be determined by the modified Omori law which can be written as

$$\Psi(t) = \frac{\theta c^\phi}{(t + c)^{1/\mu}} H(t),$$

where $c$, $\theta > 0$ and $H(t)$ is the Heaviside step function. Note that the exponent $1 + \phi$, which describes the time distribution of the direct aftershocks, is typically larger than the observed Omori exponent, which characterizes the whole cascade of directly and indirectly triggered aftershocks [Helmstetter and Sornette, 2002b; Marsan and Lengliné, 2008]. Finally, $\Phi_m(\vec{r})$ is assumed to follow the recently established epicenter distribution of aftershocks [Helmstetter and Sornette, 2002b; Felzer and Brodsky, 2006] which can be expressed as

$$\hat{\Phi}_m(\vec{r}) = \int_0^{2\pi} d\phi |\vec{r}| \Phi_m(\vec{r}) = \frac{\mu}{d(m)/d(m) + 1)^{1/\mu}},$$

where $\mu = 0.35$ and $d(m) = d_010^{0.45m}$ with $d_0 = 15$ m is the rupture length of the triggering event of magnitude $m$ [Wells and Coppersmith, 1994; Kagan, 2002; Davidsen et al., 2008]. Note that the results by Felzer and Brodsky [2006] also suggest that $\hat{\Phi}_m(\vec{r})$ does not depend on the fault orientation and, thus, the polar angle $\phi$. While we assume such a rotationally symmetric distribution of aftershocks here as well, it is important to realize that this is problematic since it neglects the typically anisotropic fault structure on which aftershocks are observed to occur and also the frequent observation that the mother event is located on the margin of the area of triggered events. Since the epicenters of the aftershock sequences considered here are approximately located on a single fault, we focus on the one-dimensional case and discuss the two-dimensional case as necessary.

The magnitude $m_i$ of each event is independently sampled from the normalized Gutenberg-Richter distribution,

$$P(m) = b \ln(10) 10^{-h(m - m_0)},$$

where the constant $b$ is typically close to 1 and $m_0$ is again the lower threshold, below which no aftershocks are initiated.
Table 1. Parameters Used for the ETAS Model

<table>
<thead>
<tr>
<th>Event</th>
<th>M</th>
<th>m0</th>
<th>$\theta$</th>
<th>c (days)</th>
<th>b</th>
<th>$\alpha$</th>
<th>K</th>
<th>$\gamma_1$</th>
<th>m2</th>
<th>$\gamma_2$</th>
<th>$\lambda_0$ (d$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parkfield</td>
<td>6.0</td>
<td>1.15</td>
<td>0.09</td>
<td>0.00395</td>
<td>0.8</td>
<td>0.2</td>
<td>0.7</td>
<td>1.2</td>
<td>1.5</td>
<td>1.5</td>
<td>1</td>
</tr>
<tr>
<td>Hector Mine</td>
<td>7.1</td>
<td>2.0</td>
<td>0.21</td>
<td>0.024</td>
<td>1.01</td>
<td>0.789</td>
<td>0.28</td>
<td>1.8</td>
<td>1.4</td>
<td>1.5</td>
<td>1</td>
</tr>
</tbody>
</table>

[9] An important quantity in the ETAS model is the average number of daughter events per earthquake, $n$, averaged over all magnitudes, which is given by

$$n = \int dx dy \int_0^{\infty} dt \int_0^{m_0} dm P(m) \phi_m(t-t_i, \vec{r} - \vec{r}_i)$$

(6)

$$n = \frac{Kb}{b - \alpha},$$

(7)

where the last equation assumes $\alpha < b$ which has been suggested as the relevant regime [Marsan and Lengline, 2008]. Note that $\vec{r} = \{x, y\}$.

2.1. Data Sets and Parameter Values

[10] Here, we study the aftershock sequences associated with the Parkfield earthquake ($M = 6.0$, 28 September 2004) and the Hector Mine earthquake ($M = 7.1$, 16 October 1999) as identified by Sheberbakov et al. [2004, 2006]. In both cases, the high-quality seismic network in the vicinity of these main shocks provided a particularly well-documented sequence of aftershocks. This is especially true for the Parkfield sequence [Bakun et al., 2005]. For Parkfield, the number of identified aftershock over a time period of $T = 365$ days is 2056 above magnitude $m = 1.15$. For Hector mine, we have 5380 aftershocks above $m = 2.0$. Both aftershock sequences differ significantly in their spatial extent and also slightly in the exponents of the Omori law and the Gutenberg-Richter law.

[11] In principle, it seems straightforward to estimate the parameters of the ETAS model for a given aftershock sequence. Yet, an important aspect of any aftershock sequence is that at early times after a main shock not all (small) aftershocks are detected due to the large amount of seismic noise [Kagan, 2004; Kagan and Houston, 2005; Peng et al., 2006]. This has led to the proposition of a time-dependent magnitude threshold of completeness which involves further parameters [Helmsateter et al., 2006]. To take this into account, we proceed as follows: First we generate artificial aftershock sequence from the ETAS model for a given main event with magnitude $M$ as outlined in section A1. Then, we impose two constraints on the generated catalog to take into account the finite area size covered by the empirical catalogs and to mimic missing data: (1) events that lie outside of the spatial area considered for the empirical catalogs were discarded and (2) events below the time-varying magnitude threshold of completeness $m_c(M, t)$ were discarded with a certain probability according to the procedure described by Lennartz et al. [2008]. To be more precise, we define $m_c(M, t) = \max[m_i(M, t), m_2]$ where $m_i(M, t) = M - 4.5 - 0.75 \log_{10}(t)$ reflects the incompleteness at time $t$ after the main event of magnitude $M$, and the time-independent sensitivity of the seismic network, different for each empirical catalog, is captured by the constant $m_2$. Then, the probability of observing an event of magnitude smaller than the threshold $m_c(M, t)$ is given by $p(m) = 10^{-\gamma(m - m_0)}$, where $j = 1$ if $m_c(M, t) = m_1(M, t)$ and $j = 2$ otherwise.

[12] Using the parameters estimated by Lennartz et al. [2008] for the Parkfield and Hector mine aftershock sequences, only $K$ and $\alpha$ of the ETAS model as well as the fraction of background events have to be approximated. Our selection criteria were both the total number of events in the catalogs, and the functional form of the total event rate (see equation (A1)). The background rate was set at 1 event/d, which gives a total event rate in very good agreement with the empirical data (not shown). The other two parameters were chosen, such that the average catalog size was similar to the empirical catalogs (after some events were discarded by the procedure outlined above), i.e., 2056 for Parkfield and 5380 for Hector Mine, with an accepted deviation of at most $\pm$ 200 events per catalog. In total, 100 artificial Parkfield and Hector Mine catalogs were generated. The parameter values used for simulating the two different aftershock sequences are summarized in Table 1.

3. Network of Recurrences

[13] We analyze the two aftershock sequences and the catalogs generated by the ETAS model according to a recently proposed method which allows one to characterize the spatiotemporal clustering of seismicity [Davidsen et al., 2006, 2008]. It is based on the notion of a recurrence in the context of spatiotemporal point processes: An event is defined to be a recurrence of a given previous event if it occurs closer in space to that event than all intervening events. Recurrences are therefore record breaking events with respect to distance. This relationship is naturally represented by a directed network of recurrences: Each event $a_k$, defined by its location $r_k$ and time of occurrence $t_k$, with $k = 1, ..., N$, is a vertex in the network and a directed edge from $a_k$ to $a_m$ exists for $k < m$ if $a_m$ is a recurrence of $a_k$, i.e., if the distance $|r_k - r_m|$ is smaller than the distance $|r_k - r_l|$ for all events $a_l$ with $k < k' < m$. This definition assumes that the events are ordered according to their occurrence in time. Each recurrence, i.e., each edge on the network, is therefore characterized by the time interval $t = t_m - t_k$ between the two connected events $k$ and $m$ and analogously by the spatial distance $l$ between the two. Note that the mapping of the point process dynamics to the recurrence network is entirely data driven and does not impose any arbitrary space and timescales other than those associated with the given event catalog.

[14] To investigate the dependence of the network properties on the magnitude thresholds of events considered, we analyze networks obtained for different magnitude thresholds. This is crucial to identify robust features as well as potential scaling properties.

4. Results

4.1. Topological Characteristics of the Network

[15] We turn now to the analysis of the statistical properties of the network of recurrences which have been proved
useful in the analysis of the overall seismic activity in southern California [Davidsen et al., 2006, 2008]. The growth of the network can be measured by the average degree $\langle k \rangle$ as a function of the number of events $N$ in the catalog. The in-(out-) degree of a node is the number of edges pointing toward (away from) it and the in- and out-degree averaged over the entire network are obviously the same. The number of nodes $N$ can be controlled by changing the magnitude threshold above which events are considered for constructing the network. As can be seen in Figure 1, the ETAS data sets as well as the empirical catalogs show a logarithmic increase of $\langle k \rangle$ with $N$. Both for Parkfield and Hector Mine, the increase is compatible with the ETAS data sets if one takes into account the statistical uncertainties. In all cases, the behavior is similar to what is expected if events were randomly, independently and uniformly placed in space and time, which would imply that the growth should follow $\langle k \rangle \sim \ln N$ for large $N$ [Davidsen et al., 2008]. Actually, this result even holds if the events are not uniformly distributed in time including the case of a simple nonhomogeneous Poisson (NHP) model following the Omori law which by construction does not have any spatiotemporal correlations between events. The comparison with such a model is particularly helpful to identify statistical properties of the empirical aftershock series and of series generated by the ETAS model that arise trivially and are, thus, not due to spatiotemporal correlations. The properties of the NHP model are analytically derived and discussed in section A2.

The probability distributions of in and out degrees can be seen in Figure 2. The out-degree distributions for both empirical and ETAS catalogs are in excellent agreement. While the distributions are compatible with a Poisson distribution over the range where we have nonzero values for the respective empirical catalog, the decay in the distribution for the respective ETAS catalog for larger values of $k_{out}$ is slightly but significantly slower than a Poisson distribution (not shown). The reason that we actually observe larger values of $k_{out}$ in the ETAS catalogs is due to the much better statistics. The situation is different for the in-degree distributions. Figure 2 shows that there are deviations between the distributions for the empirical catalog and for the ETAS catalog for both Parkfield and Hector mine. The deviations
are most pronounced for large $k_{in}$ and small $m$. In comparison to a Poisson distribution, the distributions are narrower (not shown).

[17] For the NHP model, a Poisson distribution is expected both for the in- and out-degree distributions in the limit of large networks. The observed deviations imply that the degree distributions capture some of the underlying nontrivial spatiotemporal correlations. Moreover, the differences in the in-degree distribution between the empirical data and the ETAS model are a first indication that the spatiotemporal correlations generated by the ETAS model deviate from the empirical ones.

4.2. Temporal Recurrence Statistics

[18] We turn now to the statistics of recurrence time intervals $t$, which are associated with the edges of the recurrence network. The comparison between the empirical catalogs and the ETAS model is shown in Figure 3. It can be seen that, with the parameters used for the ETAS model, and accounting properly for the missing data, the agreement between the data sets is excellent. Note that deviations for short time intervals are largely within the statistical uncertainties, but a lack of resolution and missing data in the empirical data not accounted for certainly play a role as well. This is further supported by the fact that there are basically no significant deviations for the Parkfield sequence which benefits from the higher quality of the seismic network in the region.

[19] As the insets of Figure 3 indicate, after a characteristic time, which is independent of the magnitude threshold $m$, the probability density functions (PDF) $p(t)$ of the time intervals decay as $t^{-0.9}$ and $t^{-1.1}$ for Parkfield and Hector Mine, respectively. If one does not account for missing data in the ETAS models, the exponents of the power law decay are unchanged but there are significant and systematic deviations at small timescales (not shown). In particular, the characteristic time varies with $m$. This is not only further evidence that it is crucial to take the effect of missing data into account but it also indicates that the characteristic time, which is of the order of a minute, might be an observational artifact.

[20] Further support comes from the NHP model which predicts $p(t)$ as shown in Figure 3a for Parkfield and different magnitude thresholds $m$ (see section A2 for details). Indeed, the apparent lack of variation in $p(t)$ with respect to $m$ for the NHP model is due to the dependence of the parameters in the Omori law given in equation (A2) on the magnitude threshold. In particular, the change in $c$, which could either be related to missing data or be a real effect [Shcherbakov et al., 2004, 2006; Narteau et al., 2009], prevents any significant variation in the characteristic time which would be expected otherwise (see equation (A9)). However, the NHP model also shows a faster decay than what is actually observed for larger values of $t$. In fact, equation (A9) predicts that we should not expect a pure power law for Parkfield but rather a power law with logarithmic corrections. We attribute this difference between the NHP model on one side and the ETAS model and the empirical data on the other side to the presence of spatiotemporal correlations. Clearly, this implies that at least some of the spatiotemporal correlations present in the empirical data are correctly captured by the ETAS model.

4.3. Spatial Recurrence Statistics

[21] In addition to the time interval, there is also the spatial distance $l$ associated with each edge of the network. The PDF of these distances $p(l)$ for both the empirical and ETAS catalogs are shown in Figures 4 and 5 for different magnitude thresholds $m$ and different numbers of events $N$, respectively. Contrary to the temporal statistics, there are significant differences between the ETAS model and the empirical data.

[22] For the empirical data, $p(l)$ increases for small distances up to a characteristic distance followed by a decay for...
larger arguments. (The relative location errors of the epicenters are typically between tens of meters and hundreds of meters for both aftershock sequences.) The decaying part itself consists of two regimes, a power law decay for intermediate and large distances and a much faster decay for the largest arguments. The latter is simply a finite size effect since the maximal spatial extent of the aftershock sequences is about 40 km for Parkfield and about 150 km for Hector Mine. For both Parkfield and Hector mine, the power law decay has the same exponent of about 1.05 as evident from the insets of Figures 4 and 5. The inset of Figure 4 also shows that in both cases the characteristic distance scales as $10^{0.45m}$ with magnitude threshold $m$, while it scales as $N^{-0.45}$ with the number of considered events $N$ (inset of Figure 5). Since $N \propto 10^{-m}$ according to the Gutenberg-Richter law for $b = 1$, both scaling laws are trivially related in this case. As a

**Figure 3.** Probability density functions of the time intervals between an event and its recurrences for different threshold magnitudes $m$, for (a) Parkfield and (b) Hector Mine. Solid lines represent results obtained for the ETAS model, and symbols represent for the empirical catalogs. The additional solid lines in Figure 3a correspond to curves obtained with the NHP model. The insets show the rescaled functions as indicated by the axis labels.
comparison of the insets of Figure 5 shows, the only obvious difference between the aftershock sequences of Parkfield and Hector mine are the constant prefactors in the respective scaling laws. This can be attributed to the different geometry of the faults in the areas considered. It is important to realize that all these findings are not significantly different from those for the NHP model (see equations (A7) and (A8)) suggesting that they do not reflect any nontrivial spatiotemporal correlations.

[23] For the ETAS model (see Figures 4 and 5), $p(l)$ shows a similar behavior for large and intermediate distances but there are huge deviations at length scales smaller than the characteristic distance discussed above. While there is a characteristic distance for the ETAS model, it scales as $N^{-0.8}$
and $10^{0.8m}$, respectively. (We note that in Figures 4 and 5, distances smaller than $10^{-3}$ km were not considered in order to obtain a better comparison with the empirical data sets. However, in the case of the ETAS model, this region is statistically significant, and removing it from the distribution destroys any chance of a symmetric collapse by a uniform scaling of both $x$ and $y$ axes. In our case, it suffices to scale both axes independently, where the scaling of the $x$ axis gives the characteristic length exponent, and the $y$ axis scaling is done purely in an ad hoc manner so that the curves collapse.) This is not only different from the empirical data but there is also no obvious relationship to the spatial propagation of

**Figure 5.** Recurrence distance distributions if only the first $N$ events after the main shock are considered, for (a) Parkfield with threshold magnitude $m = 1.15$ and (b) Hector Mine with $m = 2$. Solid lines represent the respective ETAS model, and symbols correspond to the empirical catalogs. The insets show the rescaled distributions, with $\gamma_x = \gamma_y = 0.45$ for both empirical data sets, $\gamma_x = 0.8$ for both ETAS data sets, $\gamma_y = -0.1$ for the ETAS model of the Parkfield sequence, and $\gamma_y = -0.15$ for the ETAS model of the Hector Mine sequence.
activity in the ETAS model defined by equation (4). Moreover, \( p(l) \) does not increase for small distances but is clearly constant. Some of these differences can be reasonably attributed to the different dimensionality of the data sets: We have assumed for the ETAS model that the epicenters of the events are displaced in one dimension, in order to mimic the displacement along a single fault. However, this assumption breaks down exactly at small distances, since on those scales the higher-dimensional structure of the fault itself becomes important. To address this point, we have analyzed a version of the ETAS model for Parkfield where the events are distributed in a two-dimensional area. While this change in dimensionality does not affect the overall functional form of \( p(l) \), the characteristic distance now scales as \( N^{-0.45} \) and \( 10^{0.45n} \), respectively. Despite this agreement with the empirical data, the prefactor in the scaling of the characteristic distance is more than an order of magnitude larger. It is also important to note that \( p(l) \) remains constant for small distances. This indicates that the spatial version of the ETAS model considered here is not able to reliably recover the spatial distribution of aftershocks on small scales. This is not unexpected since the isotropic spatial distribution of triggered events used in the ETAS model does not account for the orientations of the different faults and rupture areas. Surprisingly, however, the functional form of \( p(l) \) and the scaling behavior of the characteristic distance observed for the empirical data can be well reproduced by the NHP model (see equations (A7) and (A8)) despite an isotropic spatial distribution of events in two dimensions. This is an indication that not necessarily the isotropy of the normalized spatial distribution of directly triggered events given in equation (4) is the issue but rather those spatiotemporal correlations that affect short spatial scales.

5. Discussion

[24] It is interesting to compare the properties of the network of recurrences for the aftershock sequences associated with the Parkfield earthquake (28 September 2004) and the Hector Mine earthquake (16 October 1999) with those found for a 20 year catalog from southern California [Davidson et al., 2008]. For example, the out-degree distributions are clearly different: While they seem to follow a Poisson distribution for the isolated aftershock sequences, the distributions are significantly broader for the 20 year catalog. This indicates that spatiotemporal correlations exist between earthquakes that extend beyond what is typically considered an aftershock sequence. Another important finding is that the distributions of the time intervals between events and their recurrences are basically indistinguishable. Since the analysis of the ETAS model indicates that the absence of any dependence on the threshold magnitude for the aftershock sequences is a consequence of missing data, one can draw an analogous conclusion for the 20 year catalog. Finally, it is important to realize that the distributions of the spatial distance between an earthquake and its recurrences for the aftershock sequences do not show any indication of a nontrivial scaling with threshold magnitude. This is in sharp contrast to the 20 year catalog, which provided further evidence for a shadowing effect associated with smaller earthquakes [Rubin, 2002; Fischer and Horálek, 2005; Hainzl and Marsan, 2008]. However, the absence of a shadowing effect might be attributed to the fact that the main events of both aftershock sequences studied here were of large magnitude, for which stress shadows are seldom observed [Helmstetter et al., 2005; Main, 2006], and, hence, dominated the statistical behavior.

[25] The comparison of the nontrivial statistical features of the network of recurrences for the aftershock sequences with those for the ETAS model showed that most of them are indeed correctly captured by the model if the effect of catalog incompleteness is taken into account. This is a clear indication that a significant amount of the observed spatiotemporal correlations associated with aftershock sequences can be well understood within the framework of the stochastic model. In particular, it suggests that these spatiotemporal correlations are simply a consequence of a few empirical laws of aftershock sequences, including the Gutenberg-Richter law and the Omori law, and catalog incompleteness. Yet, the significant differences in the spatial statistics of recurrences suggest that the simple reductionist approach of the ETAS model is inadequate to explain all observed spatiotemporal correlations in aftershock sequences. Therefore, a complete stochastic description of aftershock sequences, if it exists at all, would likely have to go beyond a simple combination of independent spatial and temporal contributions to the occurrence rate, in order to be fully compatible with empirical observations. As follows from the analysis presented in this article, the statistical properties of the network of recurrences can serve as a valuable benchmark test for the spatiotemporal correlations generated by such models of aftershock sequences. This confirms earlier results for a conceptual model of earthquake dynamics [Peixoto and Davidsen, 2008]. It remains to be seen whether other stochastic models of seismicity pass this test [Vere-Jones, 2005; Turcotte et al., 2007].

Appendix A

A1. Numerical Simulation of the ETAS Model

[26] An artificial earthquake catalog can be created in an efficient manner with the ETAS model as follows. Starting from an initial event of magnitude \( M \) at the origin, and at time \( t = 0 \), the number of daughter events is sampled from a Poisson distribution with average \( \rho(M) \). For each daughter event \( i \), its magnitude, position, and occurrence time are independently sampled from \( P(m), \Phi(t_i, m) \) and \( \Psi(t_i - t) \), respectively. The same procedure is then repeated for each daughter event, recursively, until there are no more events to be generated up to a predetermined maximum time. Background events, which are not daughters of any other event, can also be added, and their daughters can be obtained in the same manner. Thus, the total number of iterations is proportional to the number of events in the final catalog. The final rate of occurrence at a given time, considering all daughter events, is then given by

\[
\lambda(t) = \lambda_b + \sum_{l \in \mathcal{L}} K 10^{a(m_l - m_0)} \frac{\theta c^g}{(t - t_i + c)}^{1/g},
\]

where \( K = n(b - \alpha)/b \), \( \lambda_b \) is the constant rate of background events, and the sum is taken over all previous events in the catalog.
A2. Simple Nonhomogeneous Poisson Model  

[27] We follow the same approach as that of Davidsen et al. [2008] which allows us to define a simple nonhomogeneous Poisson (NHP) model by specifying the functions \( \mu_t(l) \) and \( \mu_t(t, t_0) \). These functions quantify the spatial and temporal distribution of events with respect to some reference event, respectively. Note that this fact trivially excludes the existence of any spatiotemporal correlations between events. In contrast to that of Davidsen et al. [2008], we assume here that the rate of activity is not constant but given by the modified Omori law,

\[
n(t) = \frac{K}{(t + c)^p}, \tag{A2}
\]

where we restrict ourselves to the case \( p > 1 \) as observed for Parkfield [Sherbakov et al., 2006]. We note that \( p = 0 \) is not the same as \( \theta + 1 \) in equation (3) for the ETAS model, since the latter describes the rate of events directly triggered by a single event, while the former describes the total event rate. Despite this nonhomogeneous rate of activity, some of the statistical properties of the network of recurrences remain unchanged, including the topological structure, since they only depend on the spatial characteristics of the point process and the temporal ordering [Davidsen et al., 2008]. Incorporating further the effect of finite space-time domains, we have

\[
\mu_t(l) = 2a\alpha\Theta(L - l), \tag{A3}
\]

\[
\mu_t(t, t_0) = \frac{K}{(c + t + t_0)^p}\Theta(T - t - t_0). \tag{A4}
\]

\[
p_i(t) \approx \begin{cases} 
\frac{p - 1}{p^{2} - (c + T)^p} \frac{L^2aK}{2p - 1} \left( \frac{1}{c^p - 1} - \frac{1}{(c + T)^p - 1} \right) & \text{for } t \ll c \wedge T < c, \\
\frac{L^2aK}{p^{2} - (c + T)^p} \frac{L^2aK}{2p - 1} \left( \frac{1}{c^p - 1} - \frac{1}{(c + T)^p - 1} \right) & \text{for } t \ll c \wedge T < c, \\
0 & \text{for } t \gg c. 
\end{cases} \tag{A5}
\]

Here, \( a \) is some constant such that \( \mu_t(l) \) describes a homogeneous distribution of events in a two-dimensional ball of radius \( L \) around the reference event. Hence, we assume translational invariance in space. \( T \) corresponds to the finite observation period after the main shock occurring at time \( t_0 \). If the time of occurrence of the reference event. Using the formalism presented by Davidsen et al. [2008], one can compute the average PDFs of finding a recurrence at distance \( l \) from or at time \( t \) after a randomly chosen reference event. (It is important to realize that the average has to take into account that the rate of activity decays with time.) This leads to

\[
p_i(t) = \frac{(p - 1)}{p^{2} - (c + T)^p} \left( \frac{1}{c^p - 1} - \frac{1}{(c + T)^p - 1} \right) \Theta(T - t - t_0). \tag{A6}
\]

The functional behavior of these PDFs can be broken down into several parts. For \( p_i(l) \), we find

\[
p_i(l) = \begin{cases} 
\frac{L^2aK}{p^{2} - (c + T)^p} \left( \frac{1}{c^p - 1} - \frac{1}{(c + T)^p - 1} \right) & l \ll l^*, \\
\frac{2}{L^2aK} \left( \frac{1}{c^p - 1} - \frac{1}{(c + T)^p - 1} \right) & l^* \ll l < L, \\
0 & l > L,
\end{cases} \tag{A7}
\]

where the characteristic distance \( l^* \) scales as

\[
l^* \propto \left( \frac{aK}{2L^2} \left( \frac{1}{c^p - 1} - \frac{1}{(c + T)^p - 1} \right) \right)^{\frac{1}{2}} \frac{L}{\sqrt{N/2}} \tag{A8}
\]

Note that the total number of events \( N \) is simply given by \( N = aL^2 \int_0^T n(t) dt \). A comparison with the results derived in the study by Davidsen et al. [2008] for a homogeneous rate of activity shows that the qualitative features and the scaling exponents are unchanged. For \( p_i(t) \), we have

\[
p_i(t) \approx \frac{(p - 1)^2}{p^{2} - (c + T)^p} \left( \frac{1}{p^{2} - (c + T)^p} \left( \frac{1}{c^p - 1} - \frac{1}{(c + T)^p - 1} \right) \right) \Theta(L - l), \tag{A9}
\]

\[
p_i(t) \approx \frac{(p - 1)^2}{p^{2} - (c + T)^p} \left( \frac{1}{p^{2} - (c + T)^p} \left( \frac{1}{c^p - 1} - \frac{1}{(c + T)^p - 1} \right) \right) \Theta(L - l), \tag{A10}
\]

where \( \Gamma (\cdot) \) denotes the incomplete Gamma function. (The above approximation for \( t \gg c \) is based on splitting the integral into two parts, \( \int_0^T = \int_0^{t_0} + \int_{t_0}^{\infty} \). The first term was then approximated assuming \( (c + t_0) \ll t \) and taking only terms of first order for \( (c + t_0) < t \) into account. The second term was approximated assuming \( (c + t_0) \gg t \), again taking only first-order terms of \( t \) into account.) Note that depending on the exact parameter values the \( 1/t^\gamma \) regime might be present or not. (There are more cases that can be considered for \( p_i(t) \), especially one for
\[ t > (T + c)^2, \] which describes a sharp cutoff near \( t \). This case, however, only occurs over 1 order of magnitude and is therefore not important for a scaling comparison with empirical data.) For the aftershock sequence of Parkfield, one can also obtain \( p(t) \) by numerically integrating equation (A6). The values of the constants in equations (A3) and (A4) can be derived from the values estimated for the Omori law given by Shcherbakov et al. [2006]. Note that the values of \( c \) and \( K \) vary with the magnitude threshold \( m \). The variation in \( c \) can arise due to missing data (see Shcherbakov et al. [2004, 2006], Narteau et al. [2009], however, for different interpretations). For \( m = 2.15 \), we have \( c = 0.016 \) d and \( aL^2K = 21 \) d\(^{-1}\). For \( m = 1.15 \), we have \( c = 0.08 \) d and \( aL^2K = 200 \) d\(^{-1}\). These curves are plotted in Figure 3.

For \( T \rightarrow \infty \) and \( L \rightarrow \infty \), equations (A6) and (A9) reduce to

\[
p_t(t) = (p - 1)e^{p-1} - \int_0^\infty \frac{p - 1}{(c + t_0)(c + t + t_0)^p - (c + t)_p}(c + t + t_0)dt_0.
\]

(A11)

\[
p_t(t) \approx \left\{ \begin{array}{ll}
\frac{1}{t} & t < t^* \\
\frac{(p-1)^2e^{p-1}}{p} & t > t^*,
\end{array} \right.
\]

(A12)

where \( t^* \propto c \).

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References


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