Inversion of thermochronological age-elevation profiles to extract independent estimates of denudation and relief history – I: Theory and conceptual model

Pierre G. Valla a,⁎, Frédéric Herman b, Peter A. van der Beek a, Jean Braun a

a Laboratoire de Géodynamique des Chaînes Alpines, Université Joseph Fourier, BP 53, 38041 Grenoble, France
b Geologisches Institut, ETH Zürich, 8092 Zürich, Switzerland

A R T I C L E   I N F O

Article history:
Received 18 September 2009
Received in revised form 15 April 2010
Accepted 16 April 2010
Available online 18 May 2010
Edited by T.M. Harrison

Keywords:
low-temperature thermochronology
age-elevation profiles
denudation rate
relief change
inverse modelling

A B S T R A C T

We determine to what extent low-temperature thermochronology data, in particular from age-elevation profiles, provide independent and quantitative estimates on denudation rates and relief development. Thermochronological age-elevation profiles have been widely used to infer exhumation histories. However, their interpretation has remained inherently one-dimensional, neglecting potential effects of lateral offsets between samples. Furthermore, the potential effects of transient topography on crustal isotherms and consequently on thermochronological data have not yet been addressed in detail. We investigate this problem with the aim of deriving independent estimates of both denudation rates and relief history from low-temperature thermochronometers, measuring the relative uncertainties on these parameters and finally constraining the timing of potential variations in denudation rate and/or relief development. We adopt a non-linear inversion method combining the three-dimensional thermal-kinematic model Pecube, which predicts thermal histories and thermochronological ages from an input denudation and relief history, with an inversion scheme based on the Neighbourhood Algorithm. We use synthetic data predicted from imposed denudation and relief histories and quantitatively assess the resolution of thermochronological data collected along an age-elevation profile. Our results show that apatite fission-track (AFT) ages alone do not provide sufficient quantitative information to independently constrain denudation and relief histories. Multiple thermochronometers (apatite (U-Th)/He (AHe) ages and/or track-length measurements combined with AFT ages) are generally successful in constraining denudation rates and timing of rate changes, the optimum combination of thermochronometers varying with the input scenario (relief change or varying denudation rates). However, relief changes can only be quantified and precisely constrained from thermochronological age-elevation profiles if the rate of relief growth is at least 2–3 times higher than the background denudation rate. This limited resolution is due to the depth of the closure isotherm (between ∼70 and 110 °C) for the AFT and AHe systems, which only partly record topographic change. New thermochronometers (such as ⁴He/³He or OSL) that are sensitive to lower temperatures may be the key for resolving this issue.

© 2010 Elsevier B.V. All rights reserved.

1. Introduction

Understanding the formation and evolution of orogenic topography requires a better comprehension of the couplings between climate, tectonics and surface processes (e.g., Beaumont et al., 1992; Willett, 1999; Zeitler et al., 2001). However, direct evidence of these couplings remains elusive and quantitative data are needed to better constrain the denudation and relief evolution of mountain belts. These are traditionally studied independently and only few methods such as low-temperature thermochronology (e.g., Gallagher et al., 1998; Braun, 2005; Reiners and Brandon, 2006; Reiners, 2007) may enable us to quantitatively assess both the denudation history and paleo-relief of mountain belts (Braun, 2002a,b).

Here, we explore to what extent low-temperature thermochronology (apatite fission-track and (U-Th)/He) data, in particular from age-elevation profiles, can provide such constraints on denudation history and paleo-relief of mountain belts. Apatite fission-track (AFT) age-elevation profiles, i.e. sets of AFT ages collected from different elevations within a spatially restricted domain, have been widely used to infer exhumation histories of specific areas (e.g., Wagner and Reimer, 1972; Hurford, 1991; Fitzgerald et al., 1995). More recently, apatite (U-Th)/He (AHe) data have also been used in similar sampling schemes (House et al., 1997; Reiners et al., 2002; Clark et al., 2005). However, age-elevation relationships (AER) have generally considered the problem as one-dimensional, neglecting horizontal offsets between different samples of an elevation profile and therefore the potential effects of topography (Braun, 2002a; Gallagher et al., 2005).
Foeken et al., 2007). Topographic effects on thermochronological age-elevation profiles result from the fact that near-surface isotherms are not horizontal but are deflected by the topography (e.g., House et al., 1998; Braun, 2002a). The intensity of this perturbation and its depth of penetration are governed by the amplitude and wavelength of the topography. The influence of temporally steady-state topography on thermochronological age-elevation profiles is relatively well understood (Stüwe et al., 1994; Mankelow and Grasemann, 1997). However, the potential effects of temporally varying topography have not yet been addressed, although Braun (2002a) has shown conceptually what kind of patterns can be expected in AER and has highlighted the potential misinterpretation of AER slopes in terms of long-term denudation rates.

This study thus aims at: (1) deriving quantitative and independent estimates of both denudation and relief histories (rate and timing) from AER and (2) measuring the relative uncertainties on these parameters and thus the resolution of the data, which is very difficult to determine with current methods. Recent studies (e.g., Braun and van der Beek, 2004; Braun and Robert, 2005; Herman et al., 2007) have tackled similar problems by combining the three-dimensional thermal-kinematic model Pecube (Braun, 2003), which predicts thermal histories and thermochronological ages from an input denudation and relief history, with an inversion scheme based on the Neighbourhood Algorithm (Sambridge, 1999a) to search the parameter space and extract best-fitting scenarios for denudation and relief histories from the data. We adopt a similar method here but (1) specifically address the problem of interpreting age-elevation profiles and (2) add a model-appraisal stage (Sambridge, 1999a; Herman et al., in press) to fully resolve the inverse problem and derive quantitative measures of the resolution with which we infer denudation and relief histories.

Here, we use synthetic data predicted from imposed denudation and relief histories, enabling us to quantitatively assess what constraints on denudation and relief history we can extract from low-temperature thermochronology data. In a companion paper (van der Beek, P.A., P.G. Valla, F. Herman, J. Braun, C. Persano, K.J. Dobson, and E. Labrin, Inversion of thermochronological age-elevation profiles to extract independent estimates of denudation and relief history — II: Application to the French Western Alps. Submitted to Earth Planet. Sci. Lett., hereafter referred to as van der Beek et al., submitted), we apply the same methodology on a real age-elevation profile in order to investigate the potential of AER in constraining the recent evolution of the western Alps in terms of temporally varying denudation rates and/or relief development (e.g., Glotzbach et al., 2008; Vernon et al., 2009).

In the following, we first outline our conceptual model and present the synthetic data we will use in subsequent inversions. We then introduce our numerical approach and show to what extent predictions of denudation rates, timing and relief evolution can quantitatively be estimated from AER. We finally discuss the implications of our results for the use of thermochronology data in constraining tectonic and climatic controls on the evolution of orogenic topography.

2. Conceptual model

2.1. Modelling approach

In our conceptual model, outlined in Fig. 1, local exhumation rates at any point in the landscape result from two independent processes: regional denudation (which in the case of steady-state topography is equal to rock uplift as defined by England and Molnar, 1990) and temporal changes in topography. As thermochronology does not provide any constraints on surface uplift, topographic changes are expressed as changes in relief, defined as the difference in elevation $\Delta h$ between the highest and lowest points in the area under consideration. A temporal change in local exhumation rate at any point, as recorded by a thermochronometer, may thus result from any combination of varying regional denudation rates ($E_1 \rightarrow E_2$; Fig. 1a) and/or varying relief ($\Delta h_1 \rightarrow \Delta h_2$; Fig. 1b).

We use a numerical approach with simple and imposed exhumation scenarios to assess to what extent both denudation rates and relief development can be constrained independently from thermochronological data. Based on this conceptual model (Fig. 1), we predict thermochronological ages for samples along an age-elevation profile spanning the ridge-to-valley relief, for three different end-member models: (1) both denudation rate and topography are constant through time; (2) topography is steady-state but denudation rates increase through time; and (3) background denudation rates are constant but relief increases through time. In the latter case, we assume that relief increase results from preferential valley incision; i.e. ridges remain at a constant elevation with respect to an exterior reference frame but valley bottoms are lowered, implying spatially varying exhumation rates with higher rates in the valleys than on the ridges (Fig. 1b).

We compute the thermal structure through time and thermal histories for material points now at the surface using the three-dimensional thermal-kinematic model Pecube (Braun, 2003); a finite element code that solves the heat transfer equation (Carslaw and Jaeger, 1959) in 3D (Fig. 2):

$$\rho c \left( \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial z} \right) = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + H$$

(1)

where $T(x,y,z,t)$ is temperature ($^\circ$C), $\rho$ is rock density (kg m$^{-3}$), $c$ is heat capacity (J kg$^{-1}$ K$^{-1}$), $v$ is the vertical velocity of rocks with respect to the base of the model (mm yr$^{-1}$); $k$ is conductivity (W m$^{-1}$ K$^{-1}$) and $H$ is radioactive heat production (W m$^{-3}$). Pecube is able to solve Eq. (1) for a time-varying surface topography, thus permitting to model transient relief. Relief evolution is incorporated here using the relief factor $R$, defined as the ratio between the initial ($\Delta h_1$) and present-day relief ($\Delta h_2$):

$$R = \frac{\Delta h_1}{\Delta h_2}$$

(2)

Fig. 1. Conceptual model showing two end-members exhumation scenarios. Samples (black dots) are collected along an age-elevation profile spanning the ridge-to-valley relief; their thermochronological ages reflect, to a first order, the time since they passed the closure isotherm for the system considered (dashed line). (a) Increase in exhumation rate from an initial rate $E_1$ to a final rate $E_2$, under constant relief. (b) Regional exhumation rates remain constant ($E_1$) but ridge to valley relief increases from $\Delta h_1$ to $\Delta h_2$ through time. We assume relief increase to take place by valley incision; the amount of relief increase is quantified by the parameter $R = \Delta h_1/\Delta h_2$. 
If $R = 0$, there is no initial relief (i.e. the initial landscape is a plateau at the maximum present-day elevation); if $R < 1$ paleo-relief is lower than today (e.g., Fig. 1b); if $R = 1$ relief is constant through time (e.g., Fig. 1a); finally, if $R > 1$ paleo-relief is higher than at present. Note that this approach supposes that the planform pattern of ridges and valleys is fixed, only the relief changes with time.

Predicted thermal histories are used to calculate AFT and AHe age–elevation profiles; these are currently the most commonly used thermochronometers for assessing late-stage exhumation and relief development in mountain belts. Thermal histories are translated into AHe ages using a simple forward model for He production–diffusion–ejection (Farley, 2000). AFT ages are calculated using a forward model for AFT annealing (Green et al., 1989) with a new parameter fit to the laboratory annealing data (MAP3; Stephenson et al., 2006), which takes into account uncertainties in laboratory annealing temperatures as well as independent geological data. More elaborate forward models have recently been developed for both fission-track annealing (e.g., Ketcham, 2005) and He diffusion in apatite (Shuster et al., 2006; Flowers et al., 2009; Gautheron et al., 2009), which would predict slightly different effective closure temperatures depending on...
exhumation scenarios. However, forward models used for AFT and AHe age predictions are similar for both generating the synthetic data and resolving the inverse problem; the results shown here are thus largely independent of the age-prediction models we use in Pecube.

2.2. Age-elevation relationships

We use a synthetic relief model characterised by a 2D sinusoidal topography with a wavelength of 20 km and amplitude of 4 km (Fig. 2a). These values were chosen to allow sufficient denudation of valley samples for models with relief increase only; they are also characteristic of the topography observed in the western Alps study area of the companion paper. Such topography strongly perturbs low-temperature isotherms and specifically the effective Aft and AHe closure isotherms (respectively ~70 °C and ~110 °C; cf. Fig. 2a). All models are run over 30 Myr, with a late-stage change in exhumation rate or relief (Fig. 2b–e); all samples are exhumed from temperatures above the Aft and AHe closure temperature.

Fig. 2b and c shows synthetic AER for scenarios with steady-state topography and varying denudation rates (Model 1). In all cases, denudation rates are 1000 mMyr$^{-1}$ since 4.8 Ma; denudation rates prior to that time vary between 0 and 1000 mMyr$^{-1}$ for the different models. The two end-member scenarios (i.e. constant denudation rate versus discrete denudation event; respectively Model 1a and Model 1e) have widely been discussed previously in the literature (e.g., Fitzgerald et al., 1995; Gallagher et al., 1998; Braun et al., 2006); we will start by discussing these end-member cases since they provide a framework in which to discuss the other model results.

The constant-rate denudation case (Fig. 2, Model 1a) leads to perfectly linear AFT and AHe AER ($r^2=1.00$), the slopes of which, however, are 1430 mMyr$^{-1}$ and 1740 mMyr$^{-1}$ respectively (i.e., they overestimate the denudation rate by 40% to 70%) due to the perturbation of isotherms under the sinusoidal topography (e.g., Stiuwe et al., 1994; Manktelow and Grasemann, 1997). The opposite case where denudation only occurs since 4.8 Ma (Fig. 2, Model 1e) leads to AFT AER containing both a convex-up and concave-up break-in-slope (a convex-up break-in-slope only for the AHe AER). These breaks-in-slope represent the base and top, respectively, of the pre-exhumation AFT partial annealing zone or AHe partial retention zone and provide valuable constraints on the timing of onset and the total amount of exhumation (Fitzgerald et al., 1995, 1999; Gleadow and Brown, 2000).

Intermediate (and possibly more realistic) cases (Fig. 2, Models 1b, c, d), where denudation rates increase at 4.8 Ma but were non-zero before that time, lead to more-or-less linear to broadly convex-up AER from which it would be difficult to intuitively infer a sudden increase in denudation rates, especially when relative errors in AFT or AHe ages of the order of 10% are taken into account. Thus, even a five-fold increase in denudation rate (from 200 to 1000 mMyr$^{-1}$, Model 1c, Fig. 2b) may go unnoticed with AFT data only; the AER predicted by this scenario would most probably be interpreted as indicating constant exhumation at a rate ~550 mMyr$^{-1}$ ($r^2=0.95$ for a linear fit). This “mean” exhumation rate integrates exhumation at a rate of 1000 mMyr$^{-1}$ during the last 5 Myr after initial exhumation at 200 mMyr$^{-1}$. The AHe AER (Fig. 2c, Model 1c) better reflects the recent denudation history, with a linear slope of ~1300 mMyr$^{-1}$ that overestimates the true denudation rate by ~30%, but contains no record of the previous lower exhumation rates. Of course, additional constraints can be placed on the denudation histories by measuring confined fission-track length distributions (Gallagher et al., 1998; Gleadow and Brown, 2000). The above models, however, predict little variation in mean confined track lengths (MTL) for the different samples and scenarios; MTL is typically in the range 10.5–12 μm regardless of the pre-4.8 Ma denudation rate, except for very slow rates (i.e., ≤200 mMyr$^{-1}$). The latter scenarios predict a decrease in MTL with age and elevation, typical of age-elevation profiles containing an exhumed AFT partial annealing zone (Fitzgerald et al., 1995, 1999; Gleadow and Brown, 2000).

Models including relief growth (Model 2) are illustrated in Fig. 2d and e. For these models, valleys are supposed to have been carved since 4.8 Ma from a pre-existing horizontal plateau (i.e., R = 0, see Eq. (2)). This relief-growth model is combined with constant denudation, at rates between 0 and 1000 mMyr$^{-1}$, over the modelled 30 Myr timespan (Fig. 2d,e; Model 2a to 2e). Resulting AFT and AHe AER are close to linear for denudation rates ≥200 mMyr$^{-1}$ ($r^2=0.95$ for linear fits to the predicted age-elevation profiles). The AER slopes overestimate the input regional denudation rates; for instance, the slope is 230 mMyr$^{-1}$ (AFT) or 310 mMyr$^{-1}$ (AHe) for an input rate of 200 mMyr$^{-1}$; but 1840 mMyr$^{-1}$ (AFT) or 2310 mMyr$^{-1}$ (AHe) for an input rate of 1000 mMyr$^{-1}$. The reason for this overestimation is twofold. First, exhumation rates are not constant along the profile but vary from the background denudation rate at the summits to that rate augmented by the rate of valley carving (~830 mMyr$^{-1}$) at the valleys bottoms. On the other hand, since there is no relief before 4.8 Ma, isotherms are horizontal until that time and only become disturbed as valleys are carved, thus limiting the topographic effects on age-elevation profiles. For moderate to high background denudation rates, isotherms are advected to the surface, allowing to better record post-4.8 Ma relief development. However, for denudation rates <200 mMyr$^{-1}$, heat advection is minor and erosion in the valleys is insufficient to expose samples exhumed from below the AFT partial annealing zone (for rates <100 mMyr$^{-1}$ the AHe partial retention zone is not exposed either), leading to distinctive concave-up AER. Such profiles might be interpreted as representing exhumation at rates that diminish through time or, if track-length data are taken into account, as limited and slow long-term denudation (e.g., Glatzach et al., 2008; Reiners and Brandon, 2006).

Differences between age-elevation profiles predicted by very different scenarios may be subtle and the predicted patterns counter-intuitive. As shown in Fig. 2, scenarios with (Model 2) or without (Model 1) topographic change can be interpreted in the same way, judging from the pattern and slope of the AER. As discussed above, implicitly one-dimensional interpretation of such profiles is limited because of horizontal offsets between profile samples (up to 10 km in our simulations, Fig. 2) and topographic disturbance of isotherms. For example, Model 1c simulates an increase in denudation rate from 200 mMyr$^{-1}$ to 1000 mMyr$^{-1}$ with constant topography; whereas Model 2b combines topographic change with a regional exhumation rate of 500 mMyr$^{-1}$. Although these two scenarios are completely different, AFT AER (Fig. 2d) have indistinguishable slopes of ~660 mMyr$^{-1}$ (Model 1c) and ~700 mMyr$^{-1}$ (Model 2b) respectively. AHe patterns (Fig. 2c,e) are slightly different (~1300 mMyr$^{-1}$ for Model 1c and ~1000 mMyr$^{-1}$ for Model 2b); however the difference remains subtle and does not resolve the input denudation and relief histories. Straightforward interpretation of these two age-elevation profiles will lead to inferred spatially and temporally averaged exhumation rates that have no simple relation to the true exhumation history and do not address potential topographic changes.

3. Inversion method and results

3.1. Methodology: Pecube coupled to the Neighbourhood Algorithm (NA)

We choose two end-member scenarios to test whether numerical inversions can fully retrieve denudation and relief histories from AER (Fig. 2, respectively Models 1c and 2c): (1) constant topography (R = 1) but an abrupt change in denudation rates ($E_1 = 200$ mMyr$^{-1}$; $E_2 = 1000$ mMyr$^{-1}$) 4.8 Ma ago (referred to in the following as Scenario 1); and (2) a constant denudation rate $E_1 = E_2 = 200$ mMyr$^{-1}$ but 4 km of relief growth ($R = 0$) since 4.8 Ma (Scenario 2). Thermo-kinematic data used as input parameters in
Pecube are given in Table 1; the simulation was run over 30 Myr to ensure that samples are exhumed through their closure isotherm and that isotherms are at steady state before increasing denudation or relief. Fig. 3 shows the synthetic datasets used for inverse modelling; they consist of AFT and AHe ages as well as mean fission-track lengths (MTL). AER for AHe and AFT are quite similar for the two scenarios except that the age contrast between ridge and valley samples is higher for Scenario 2. The main difference between the two scenarios is found in the MTL pattern, with an inverse (Scenario 1) versus normal (Scenario 2) correlation between MTL and elevation.

Using these predicted “perfect” data, we explore for both Scenarios 1 and 2 what quantitative constraints can be set on denudation rates, timing and relief evolution. To do this, we first define for each parameter a specified range that will be searched by the inverse algorithm (uniform distribution for the priors): for the denudation rate of the first phase \((E_1): 0–1000 \text{ m Myr}^{-1}\); denudation rate of the second phase \((E_2): 0–1000 \text{ m Myr}^{-1}\); transition time \((T): 0–30 \text{ Ma}\); and relief factor \((R): 0–2\). Since we wish to test to what extent a specific thermochronological dataset can provide independent estimates on those four parameters, we have run four inverse simulations (Table 2) for both Scenarios 1 and 2 using: (1) AFT ages only (inversions 1-A and 2-A); (2) AFT + AHe ages (inversions 1-B and 2-B); (3) AFT ages + MTL (inversions 1-C and 2-C); and finally AFT + AHe + MTL (inversions 1-D and 2-D).

The Neighbourhood Algorithm (NA) is a two-stage numerical approach to derive Bayesian estimates on input parameters for non-linear inverse problems (Sambridge, 1999a,b). The first or sampling stage is an iterative search method during which sampling gradually concentrates on regions of the multidimensional parameter space where the misfit function is optimised (i.e. sets of parameters values

### Table 1

Thermo-kinematic and elastic parameters used in Pecube. Crustal thickness and basal temperature are set to obtain a geothermal gradient of 25 °C km\(^{-1}\). Poisson ratio, Young’s modulus and equivalent elastic thickness are used for calculating the isostatic rebound in response to relief change. Equivalent elastic thickness is set to a value that simulates moderate isostatic rebound.

<table>
<thead>
<tr>
<th>Parameter (unit)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crustal density (kg m(^{-3}))</td>
<td>2700</td>
</tr>
<tr>
<td>Sublithospheric mantle density (kg m(^{-3}))</td>
<td>3200</td>
</tr>
<tr>
<td>Equivalent elastic thickness (km)</td>
<td>25</td>
</tr>
<tr>
<td>Young’s modulus (Pa)</td>
<td>1.10^{15}</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.25</td>
</tr>
<tr>
<td>Crustal thickness (km)</td>
<td>20</td>
</tr>
<tr>
<td>Thermal diffusivity (km(^2)Myr(^{-1}))</td>
<td>25</td>
</tr>
<tr>
<td>Basal crustal temperature (°C)</td>
<td>520</td>
</tr>
<tr>
<td>Sea-level temperature (°C)</td>
<td>15</td>
</tr>
<tr>
<td>Atmospheric lapse rate (°C km(^{-1}))</td>
<td>6</td>
</tr>
<tr>
<td>Crustal heat production (°C Myr(^{-1}))</td>
<td>0</td>
</tr>
</tbody>
</table>

![Fig. 3. Synthetic thermochronological data used for the inverse modelling.](image)
Table 2
Bayesian estimates after the NA appraisal stage for input parameters $E_i$, $E_r$, $T$ and $R$ (see text for description and discussion). First column defines the parameters for the two scenarios. Second column gives prior PDF, i.e. the PDF describing the input parameter range for the inversion. Third column reports the parameters used to calculate the input thermochronological data (Fig. 3). Optimal values and estimated uncertainties of parameters are given in following columns for all inversion simulations (inversions 1-A/2-A, 1-B/2-B, 1-C/2-C and 1-D/2-D).

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Prior Pdf</th>
<th>Input value</th>
<th>AFT (inversion 1-A/2-A)</th>
<th>AFT + AHe (inversion 1-B/2-B)</th>
<th>AFT + MTL (inversion 1-C/2-C)</th>
<th>AFT + AHe + MTL (inversion 1-D/2-D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>$E_i$ (km Myr$^{-1}$)</td>
<td></td>
<td>$1.1 \pm 0.6$</td>
<td>$0.2 \pm 0.2$</td>
<td>$0.3 \pm 0.2$</td>
<td>$0.6 \pm 0.4$</td>
</tr>
<tr>
<td></td>
<td>$E_r$ (km Myr$^{-1}$)</td>
<td></td>
<td>$1.1 \pm 0.6$</td>
<td>$1.0 \pm 0.3$</td>
<td>$0.9 \pm 0.3$</td>
<td>$0.9 \pm 0.3$</td>
</tr>
<tr>
<td></td>
<td>$T$ (Ma)</td>
<td></td>
<td>$3.2 \pm 1.1$</td>
<td>$4.3 \pm 1.2$</td>
<td>$3.2 \pm 1.2$</td>
<td>$3.2 \pm 1.2$</td>
</tr>
<tr>
<td></td>
<td>$R$</td>
<td></td>
<td>$1.0 \pm 0.6$</td>
<td>$1.0 \pm 0.6$</td>
<td>$1.1 \pm 0.5$</td>
<td>$0.7 \pm 0.4$</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>$E_i$ (km Myr$^{-1}$)</td>
<td></td>
<td>$1.1 \pm 0.6$</td>
<td>$0.2 \pm 0.2$</td>
<td>$0.2 \pm 0.2$</td>
<td>$0.2 \pm 0.2$</td>
</tr>
<tr>
<td></td>
<td>$E_r$ (km Myr$^{-1}$)</td>
<td></td>
<td>$1.1 \pm 0.6$</td>
<td>$1.0 \pm 0.3$</td>
<td>$0.9 \pm 0.3$</td>
<td>$0.9 \pm 0.3$</td>
</tr>
<tr>
<td></td>
<td>$T$ (Ma)</td>
<td></td>
<td>$3.2 \pm 1.1$</td>
<td>$4.3 \pm 1.2$</td>
<td>$3.2 \pm 1.2$</td>
<td>$3.2 \pm 1.2$</td>
</tr>
<tr>
<td></td>
<td>$R$</td>
<td></td>
<td>$1.0 \pm 0.6$</td>
<td>$1.0 \pm 0.6$</td>
<td>$1.1 \pm 0.5$</td>
<td>$0.7 \pm 0.4$</td>
</tr>
</tbody>
</table>

that minimize the misfit to the data). The parameter space is divided into Voronoi cells, centred on each sampled model, that represent the nearest neighbourhood about each point. A certain amount of best-fitting forward models (here 50%) is used to define new Voronoi cells and thus to fix a new parameter space. This new parameter space is then sampled during the next iteration in a random fashion, eventually converging towards one or several sets of parameters that minimise the misfit function to the data. In our approach, we use a weighted least-squares misfit function $\psi$:

$$\psi = \sqrt{\sum_{i=1}^{M} \left( \frac{\chi_{i,mod} - \chi_{i,dat}}{\sigma_i^2} \right)^2}$$

(3)

where $N$ is the number of datasets (1 for inversions A; 2 for inversions B and C; 3 for inversions D), $M$ is the number of samples in each dataset (11), $\chi_{i,mod}$ and $\chi_{i,dat}$ are predicted and observed values for AFT/AHe ages or MTL, respectively, and $\sigma_i$ is the uncertainty on the data. Here, we choose to use constant synthetic uncertainties ($\sigma_i$) of $\pm 0.5$ Ma, $\pm 0.3$ Ma and $\pm 0.2 \mu$m for AFT, AHe and MTL, respectively, to provide an equal weight on all samples for misfit calculations. We choose to fit the mean track length (MTL) rather than the full track-length distribution for computational convenience and because we can simply define an uncertainty on MTL that is comparable to the age uncertainty. Note that we do not use a reduced misfit, i.e. the misfit function will be higher when we use a larger number of data. Using a high performance cluster, we perform a large amount of forward model runs (5000 per inversion), necessary to obtain a statistically significant result given the number of free parameters.

During the second or appraisal stage, the algorithm resamples the entire ensemble of models that were generated during the first stage to provide Bayesian measures of resolution in the form of marginal probability density functions (PDF) $L$ for each parameter, following the equation:

$$L = \prod_{i=1}^{N} \exp \left( -\frac{1}{2} \sum_{j=1}^{M} \frac{\left( \chi_{i,mod} - \chi_{i,dat} \right)^2}{\sigma_i^2} \right).$$

(4)

3.2. NA sampling stage results

We first present the results of the sampling stage, i.e. the parameter optimisation, for Scenarios 1 (Fig. 4) and 2 (Fig. 5). Each forward model is represented by a dot, the colour of which depicts the value of the misfit function $\psi$ for the associated model (Eq. (1)), reduced by the number of data. The use of a reduced misfit function in Figs. 4 and 5 allows us to directly compare inversion runs with different numbers of data. Parameter estimates are determined graphically, arbitrarily considering models represented by misfit values $<0.3$ as “satisfying”. We only represent scatterplots for inversions 1-A/2-A (AFT ages only) and 1-D/2-D (full dataset) since they illustrate how coupling different thermochronometers may provide better predictions on input parameters. Intermediate inversion results combining AFT ages with either MTL or AHe ages will be shown for the appraisal stage.

For Scenario 1 (Fig. 4), using AFT ages only (inversion 1-A, Fig. 4a,b) provides accurate but imprecise predictions for $E_i$ ($0.2–0.5$ km Myr$^{-1}$) and $T$ ($7$–$3$ Ma). However, Run 1-A leads to erroneous estimates for both $E_i$ and $R$, suggesting a minor increase in denudation rate ($E_i \approx 0.4–0.7$ km Myr$^{-1}$) compared to the input value ($E_{\text{input}} = 1$ km Myr$^{-1}$) but converging toward significant relief increase ($R \approx 0.4$) instead of a constant topography ($R_{\text{input}} = 1$). Adding MTL and AHe ages to AFT ages (inversion 1-D; Fig. 4c,d) leads to much better predictions. The precision of the estimated $E_i$ and $T$ is improved ($E_i \approx 0.15–0.25$ km Myr$^{-1}$; $T \approx 4–6$ Ma). Moreover, estimates on $E_i$ ($\approx 0.7$–$1$ km Myr$^{-1}$) and $R$ ($\approx 0.5–0.9$) are much closer to the input values for inversion 1-D than for inversion 1-A.

We adopt the same methodology for Scenario 2 (Fig. 5), presenting first inversion results using AFT ages only (inversion 2-A; Fig. 5a,b) and then for AFT ages combined with both MTL and AHe ages (inversion 2-D; Fig. 5c,d). Using AFT ages only in Scenario 2 provides different results than for Scenario 1: both $E_i$ and $E_r$ are predicted correctly ($E_i \approx 0.15–0.25$ km Myr$^{-1}$ and $E_r \approx 0.1–0.4$ km Myr$^{-1}$) whether we use AFT ages alone (inversion 2-A) or the full dataset (inversion 2-D). The main differences between inversions 2-A and 2-D lie in the predictions for $T$, which are much less precise and accurate for inversion 2-A ($T \lesssim 5$ Ma) than for inversion 2-D ($T \approx 4–5$ Ma; $T_{\text{input}} = 4.8$ Ma). Predictions on $R$ suggest a lower relief increase than the input value ($R \lesssim 0.4$; $R_{\text{input}} = 0$) whether we use AFT ages alone (inversion 2-A) or the full dataset (inversion 2-D).

These results show that the two input scenarios (increase in denudation rate versus relief growth) do not lead to the same inversion results. Although our graphical interpretation of these scatterplots (Figs. 4 and 5) remains qualitative, these results strongly suggest that an AFT AER alone is not sufficient to provide quantitative independent estimates on denudation and relief histories; AHe and MTL datasets are needed to constrain the input scenario, leading to much better parameter estimates when using all the available data even in relatively simple cases with only a single change in relief or denudation rate.

3.2.1. NA appraisal stage results

Results from the NA appraisal stage differ from the sampling stage since we do not focus on “satisfying” models via graphical inspection but investigate the statistical properties of the model ensemble.
Posterior PDF’s of parameter values for Scenarios 1 and 2 are plotted in Figs. 6 and 7, respectively. Table 2 reports both the most probable value and the estimated uncertainty, defined respectively by the mode and standard deviation of the marginal PDF, for each parameter ($E_1$, $E_2$, $T$ and $R$) and all inversions (1-A to 1-D and 2-A to 2-D).

For Scenario 1, the different inversions (1-A, 1-B, 1-C and 1-D) do not lead to similar quantitative predictions for the 4 parameters ($E_1$, $E_2$, $T$ and $R$). $E_1$ and $T$ estimates are not better resolved than the prior estimates (i.e. a uniform distribution within the imposed limits) when using AFT ages only or AFT+AHe ages (Fig. 6a–c). Adding MTL to these datasets provides predictions that are closer to the input values and have lower uncertainties (inversions 1-C or 1-D). Including MTL thus appears to be more important than AHe ages to optimize predictions on these two parameters ($E_1$ and $T$). In contrast, accurate estimates for $E_2$ (Fig. 6b) requires AFT + AHe ages (inversion 1-B), MTL (inversion 1-C) or both (inversion 1-D), even though the resolution is similar whatever the input data. Finally, Bayesian estimates for $R$ (Fig. 6d) are poor for all runs, with high uncertainties ($\pm 50\%$) even when we combine AFT and AHe ages with MTL (inversion 1-D).

For the relief-growth experiment (Scenario 2; Fig. 7), Bayesian results show that we provide tight constrains on $E_1$ (Fig. 7a) even when using AFT ages only (inversion 2-A). Only the resolution of $E_1$ is improved by adding more information ($0.2 \pm 0.1$ km Myr$^{-1}$) for inversion 2-A compared to $0.2 \pm 0.04$ km Myr$^{-1}$ for inversion 2-D). Predictions for $E_2$ and $T$ are also quite similar for all the inversions (2-A to 2-D; Fig. 7b,c); they all underestimate the age of relief change ($T_{input} < T_{input}$) and overestimate the denudation rate ($E_2 > E_{input}$) of the second phase. Inversion 2-D, which combines AFT + AHe ages with MTL, leads to $E_2$ and $T$ predictions ($E_2 = 0.3 \pm 0.1$ km Myr$^{-1}$; $T = 4.0 \pm 1.1$ Ma) that are
closer to the input values \( E_2^{\text{input}} = 0.2 \, \text{km Myr}^{-1} \); \( T^{\text{input}} = 4.8 \, \text{Ma} \). In this case, AHe ages seem to be as important as MTL and AFT ages for providing reliable estimates of \( E_2 \) and \( T \). Finally, predictions of \( R \) (Fig. 7d) with AFT ages only \( (R = 0.3 \pm 0.2) \) or AFT + AHe ages \( (R = 0.2 \pm 0.2) \) are close to the input value \( (R^{\text{input}} = 0) \) even though they consistently underestimate the relief increase by \( \sim 20\% \). Adding MTL to AFT + AHe ages \( (\text{inversion 2-D}) \) does not improve the prediction \( (R = 0.3 \pm 0.2) \). Using AFT ages and MTL without AHe data \( (\text{inversion 1-C}) \) clearly underestimates the relief increase by \( \sim 60\% \).

Our results show that inverse numerical modelling of thermochronological AER combining various thermochronometers is an efficient tool for extracting quantitative information on denudation and relief histories. In the following, we will discuss our numerical results and their implications concerning the use of low-temperature thermochronometers for quantifying denudation histories and relief changes.

### 4. Discussion

#### 4.1. Direct interpretation of AER

Our results show that even simple tectono-morphic scenarios such as a single increase in denudation rate \( (\text{Model 1}) \) or relief \( (\text{Model 2}) \) provide AER that may be problematic to interpret at face value (Fig. 2). AER may be powerful tools to detect first-order tectonic or geomorphic changes, such as a pulse of denudation after a long period of quiescence \( (\text{Model 1e}; \text{Fig. 2b,c}) \) or relief carving with a low background denudation rate \( (\sim 200 \, \text{mMyr}^{-1}) \); Models 2d and 2e, Fig. 2d,e). This exhumation pulse is recorded by AFT and AHe thermochronometers via exhumation of the partial annealing zone \( (\text{PAZ}) \) for FT and partial retention zone \( (\text{PRZ}) \) for He, which imprints a clear signal in the AER \( (\text{e.g., Fitzgerald et al., 1995, 1999}) \). More subtle changes in denudation rates and/or relief changes do not have such a
clear signal in AER, in particular because elevation profiles are not vertical and samples are laterally offset. Furthermore, direct interpretation is difficult since AER are often close to linear; the associated slope only provides a spatially and temporally averaged exhumation rate through the sampled time window. More information can be retrieved when including AHe data; however, we have seen that except in specific cases (i.e. denudation rates \( < 200 \text{ mMyr}^{-1} \)), AHe profiles resemble AFT AER and do not provide sufficient additional information to derive independent estimates on denudation and relief histories. The usefulness of track-length measurements for constraining relief growth is limited by their correlation with elevation (Fig. 3d) that is similar to the correlation observed for denudation scenarios under steady-state topography, except where denudation rates increase by at least a factor of 5. In the latter scenarios, MTL patterns show a clear inverse correlation with elevation (Fig. 3c).

4.2. Quantitative inversion of thermochronological data: multiple datasets

Using Pecube with an inverse approach (NA algorithm), we have analysed to what extent a specific dataset can provide quantitative estimates on denudation rates, timing and relief history. Before discussing the results, we should point out some limitations associated with our modelling approach. First, we have implemented relief changes in such a way that the planform topographic pattern does not change with time. This is not a limitation of the Pecube code, which can handle any arbitrary topographic change, but permits us to simply parameterize relief change through a single parameter \( R \). We note, however, that evidence for relative longevity of drainage patterns exists in many mountain belts and present such evidence for the Western Alps study area in the companion paper. Second, the scenarios we have run, which all involve a recent increase in relief and/or denudation rate, are somewhat contextual and obviously only address a limited number of exhumation and relief scenarios from the infinite choice available. However, such recent changes in exhumation rate and/or relief in mountain belts have been investigated by many studies using thermochronological AER in recent years (e.g. Fitzgerald et al., 1995, 1999; Densmore et al., 2007; Gibson et al., 2007; Whipp et al., 2007; Glotzbach et al., 2008; Vernon et al., 2009). Third, our inferences are obviously somewhat dependent on the age-prediction model for AFT and AHe that we have chosen to implement, as well as on the fact that we only use MTL instead of the full track-length distribution.

Keeping the above caveats in mind, we focus here on predictions concerning denudation history \( (E_1 \text{ and } E_2) \) and timing \( (T) \); quantitative estimates on relief evolution \( (R) \) will be discussed in the next section. In the case of an increase in denudation rate under steady-state topography (Scenario 1), parameter estimates from AFT ages alone are poor (Fig. 6; Table 2). The inability of AFT ages alone to retrieve denudation and timing parameters explains why the inversion algorithm converges towards a parameter set that is different from the input values (Inversion 1-A, Fig. 4a,b). We have performed two tests to ascertain that it is the data and not the inversion procedure that leads to this erroneous prediction (See Supplementary Figs. 1 and 2). Resampling a much larger proportion of models between different generations leads to slower convergence but the same predicted optimal parameter values. In contrast, setting
$R = 1$ and only solving for denudation rates and timing leads to correct parameter predictions. These tests show that AFT data on their own do not have sufficient resolution to constrain the denudation and relief history and also that care should be taken when interpreting the optimisation results from NA on their own, without considering the resolution of the parameter predictions.

In contrast, AFT data alone lead to good predictions for the input parameters in a relief-growth context (Scenario 2), even though resolution is relatively low (Fig. 7). We thus suggest that, in the cases studied here, AFT data are more useful for studying relief growth than for quantifying variations in denudation rate under constant topography. This may be explained by the shape of the AER, which is less linear in the relief-growth case (Fig. 3b) and shows a higher contrast in AFT ages between ridge and valley-bottom samples than in the denudation increase scenario (Fig. 3a).

In Scenario 1, AHe ages combined with AFT ages do not add more information than AFT ages alone and predictions of all parameters except $E_2$ remain poor (Table 2). The AHe system has a lower closure temperature than AFT (respectively $\sim 75 ^\circ C$ and $\sim 110 ^\circ C$), which explains why AHe ages provide better constraints on the recent denudation history ($E_2$). Adding MTL data leads to better constraints on both exhumation rates and timing. Quantitative estimates of the exhumation history under steady-state topography thus require both AFT ages and MTL measurements, whereas AHe ages do not seem to improve parameter predictions. The reason for this lies in the AHe and MTL profiles (Fig. 3): the AHe AER is very similar to the AFT profile whereas the MTL pattern is strongly contrasted between ridge (short MTL) and valley-bottom (longer MTL) samples.

Scenario 2 (i.e., relief growth) shows completely different results. In this case, AHe ages combined with AFT ages provide much better estimates of $E_1$ and $T$ (Table 2 and Fig. 7a–c), $E_2$ being already well constrained by AFT ages alone (Fig. 7b). In contrast, using MTL measurements with AFT ages leads to both an overestimation of $E_2$ and an underestimation of $T$. This difference with Scenario 1 may be due to the MTL pattern (Fig. 3d), which is characterised by a normal correlation between MTL and elevation in all relief-growth scenarios. Moreover, the AHe AER shows a higher contrast between ridge and valley-bottom samples than the AFT AER (Fig. 3b), since AHe has a lower closure temperature and thus more potential to record changes in either exhumation rates or relief close to the surface. We thus suggest that in the case of relief change studied here, AHe data provide more information than MTL measurements.

The problem of course is that in a real-world study, one usually does not know a-priori what scenario (varying denudation rates or relief change) to expect, and this will probably be the objective of the study. Therefore, the best approach appears to be to collect as much independent thermochronometry data as possible and typically to combine AFT and AHe ages with track-length measurements along altitudinal profiles. The fundamental reason for applying multiple thermochronometers is that AER sample the topography at a single wavelength (20 km in our case), which is problematic for extracting independent information on denudation and relief evolution (Braun,
Fig. 8. Scatterplots showing the quality of relief-change predictions ($R$) by the inversion approach developed here as a function of both denudation and relief-growth rates. Each dot results from an inverse model with a specified input denudation rate and relief evolution scenario. (a) Absolute difference between predicted and “true” $R$ (the latter indicated by $R_{\text{input}}$). (b) 1σ uncertainty on predicted $R$ (defined as the standard deviation of the PDF of $R$ from its mean value). 1:1 (solid line), 1:2 (dashed line) and 1:3 (dotted line) ratio of denudation rate versus relief-growth rate are indicated (see Supplementary data for numerical results).

5. Conclusions

This study shows that thermochronological age-elevation profiles may be used to quantitatively define denudation and relief histories in certain cases. We have used the thermal-kinematic 3D model Pecube to predict AFT and AHe AER and MTL patterns from imposed input scenarios with varying denudation rates or/and relief histories. Predicted AFT AER mostly depend on the ratio between the background denudation rate and relief growth (Fig. 8). Our inversion approach leads to relatively accurate predictions on $R$ (i.e., $<$50% error between $R$ and $R_{\text{input}}$) for scenarios where the relief-growth rate is at least 2 times higher than the background denudation rate (Fig. 8a). However, obtaining satisfactory resolution on $R$ (i.e., uncertainty $<$0.4) requires a relief-growth rate that is at least 3 times higher than the background denudation rate (Fig. 8b). This sensitivity analysis suggests that relief development must be significantly more rapid than the background denudation rate to be quantitatively extracted from thermochronological AER, which strongly reduces the number of suitable orogenic settings for this kind of investigations. In effect, we suggest that this approach may be most successful in settings with low background denudation rates and potentially major recent relief increase, such as the Sierra Nevada (California) or the Pyrenees (Spain, France).

Interestingly, in both settings contrasting conclusions have been drawn concerning recent relief change from the interpretation of thermochronological AER (e.g., House et al., 2001 versus Clark et al., 2005 for the Sierra Nevada; Fitzgerald et al., 1999 versus Gibson et al., 2007 for the Pyrenees). In contrast, it does not appear very well suited to constrain potential relief changes in high-exhumation-rate settings such as the Himalaya (Whipp et al., 2007) or the Southern Alps of New Zealand (Herman et al., 2007). New thermochronometers and models based on the He distribution within single apatite grains ($^{4}$He/$^{3}$He thermochronology; Shuster et al., 2005) or OSL-thermochronometry (Herman et al., in press) may provide more precise information on relief changes, especially if they occurred during Quaternary times.
may be the key for resolving this issue as they more precisely record denudation rates and/or geomorphic changes close to the surface.

Acknowledgments

This study is supported by INSU-CNRS through the European Science Foundation Eurocores Topo-Europe programme 07-TOPO-EUROPE-FP023 “Coupled climatic/tectonic forcing of European topography revealed through thermochronometry (Thermo-Euro)” and the Agence Nationale de la Recherche project No. ANR-08-BLAN-0303-01 “Erosion and Relief Development in the Western Alps”. It forms part of PV’s PhD project at Université Joseph Fourier, supported by the French Ministry for Research and Higher Education. Computations were performed on Brutus, the high performance computing facilities at ETH Zürich. The codes are available at http://svn-geo.ethz.ch after registering at this site. Thorough and constructive reviews by Mark Brandon and Richard Ketcham significantly improved the manuscript.

Appendix A. Supplementary data


References